

Reduced basis element modeling

The reduced basis method is used for parameter dependent problems like

$$F(u; \mu) = 0,$$

where u is the unknown, and μ is some parameter. The idea is to compute a basis for the problem by solving for a set of given parameters, $S_N = \{\mu_i\}_{i=1}^N$. The resulting basis functions $X_N = \{u_i\}_{i=1}^N$ are then used to approximate the unknown solution for a new choice of the parameter,

$$u(\mu) = \sum_{i=1}^N \alpha_i(\mu) u_i$$

by determining the coefficients α_i . This is similar to the traditional finite element method, with the low order finite element basis functions replaced by global basis functions very relevant to the actual problem.



Figure 1: Solutions found on variations of the geometry on the left are used to generate the solution on the 4-stage fin on the right.

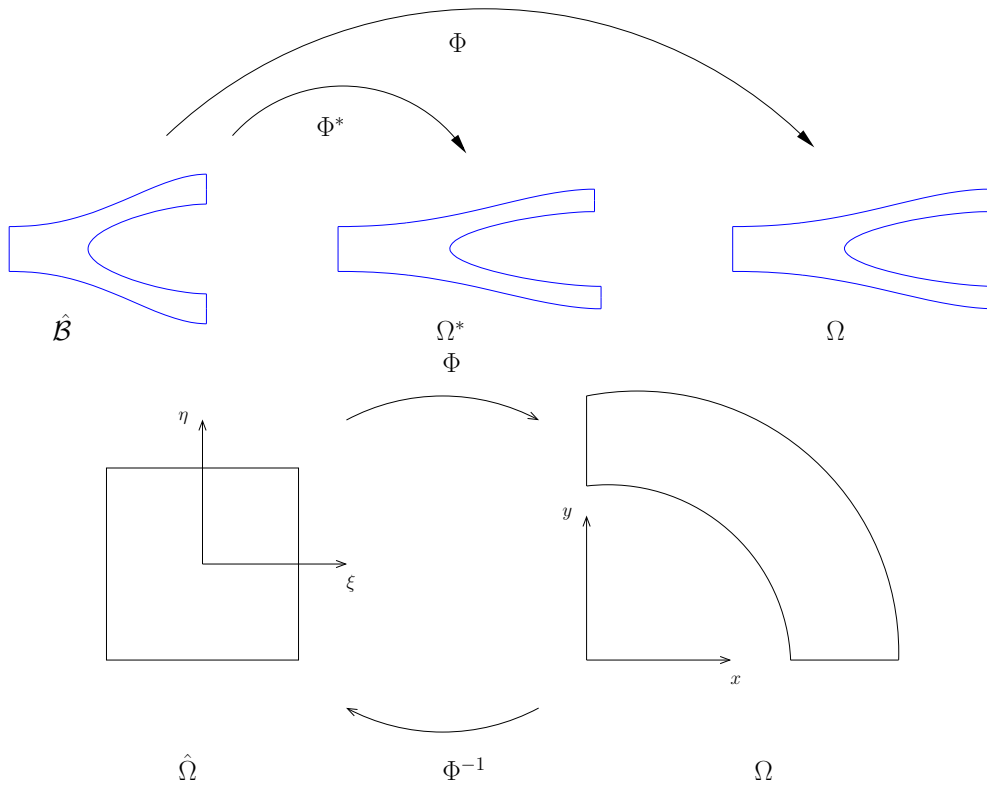


Figure 2: Example of two different reference building blocks mapped to topologically similar shapes.

In the reduced basis *element* method, we introduce the geometry of the computational domain as the parameter [1, 2]. To do this we assume that the computational domain is found as a map of a reference building block. In Figure 2 this is illustrated for two different reference building blocks. By gluing the different shapes together, we may also construct systems like the one in Figure 3.

Master project

The master project is to implement the reduced ba-

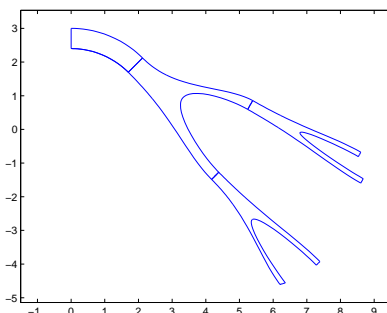


Figure 3: Example of a system glued together by different shapes.

sis element method in three dimensions. The underlying problem may be heat diffusion, like the thermal fin shown in Figure 1, or fluid flow problems. The project will give a good understanding of how geometries other than the unit square are included in numerical solution of PDEs, and there are several challenges in the implementation of the reduced basis method. The goal of the project is to use the reduced basis element method to find an approximation to the underlying problem for a large number of generic geometries, and possibly choose the optimal geometry with respect to some criterion.

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References:

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- [2] A. E. Løvgren, Y. Maday, and E. M. Rønquist. A reduced basis element method for the steady Stokes problem. *M2AN*, 40(3):529-552, 2006.